

کانون تخصصی مشاوره تحصیلی  
مهندس هادی فرقانی

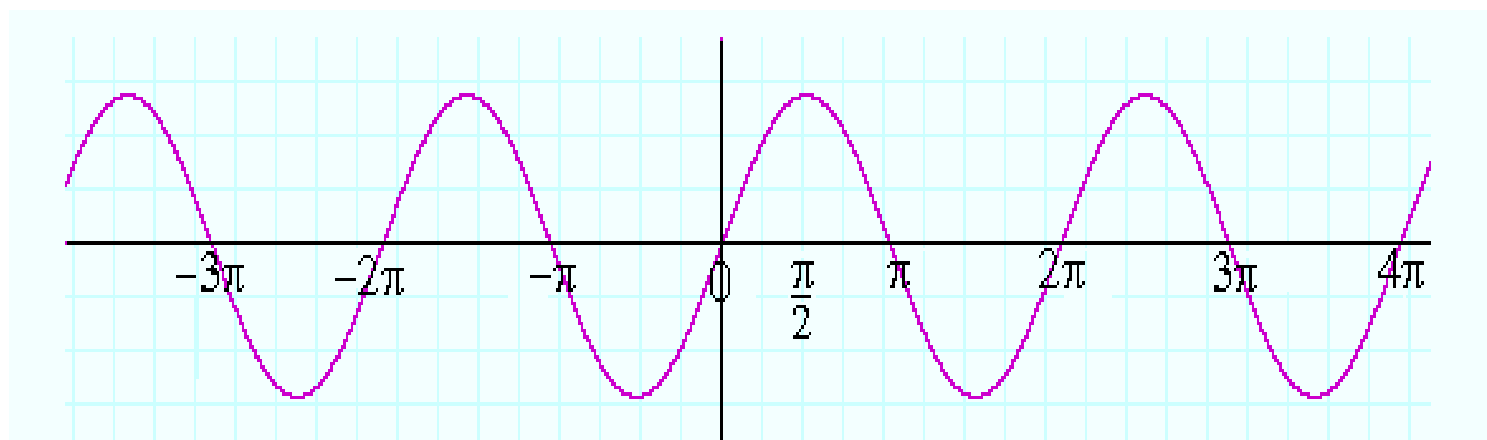


↓ کانال تلگرام ↓

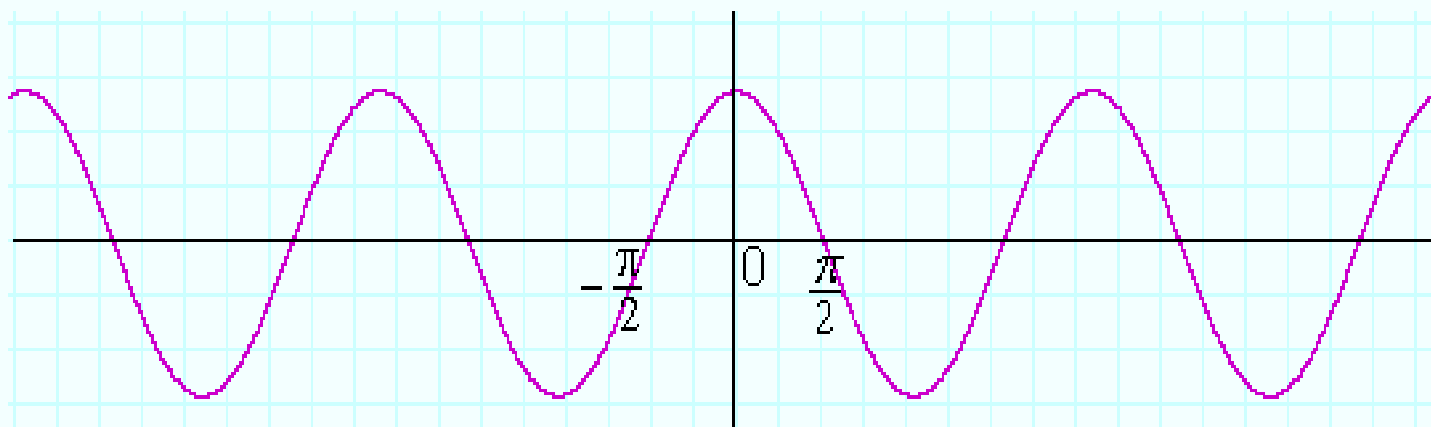
[telegram.me/moshaverekoonkour](https://t.me/moshaverekoonkour)

[www.konkor.net](http://www.konkor.net)

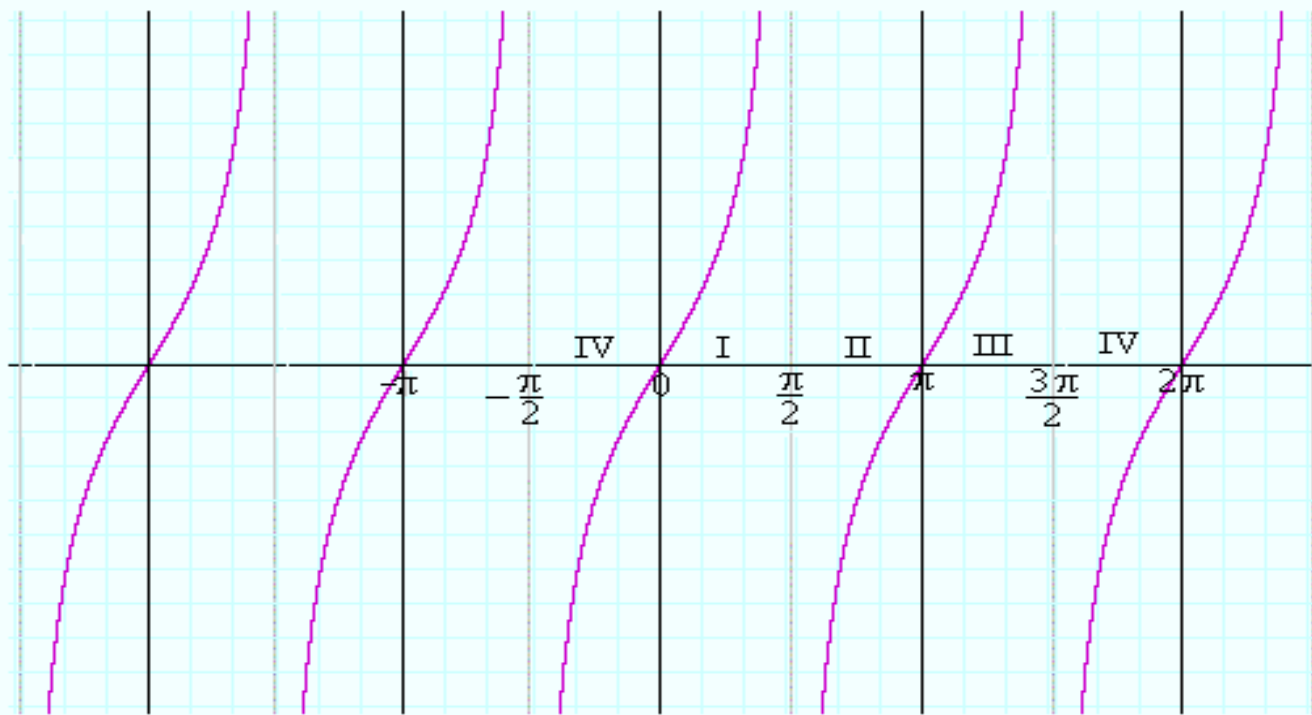
# $Y = \sin(x)$ گراف



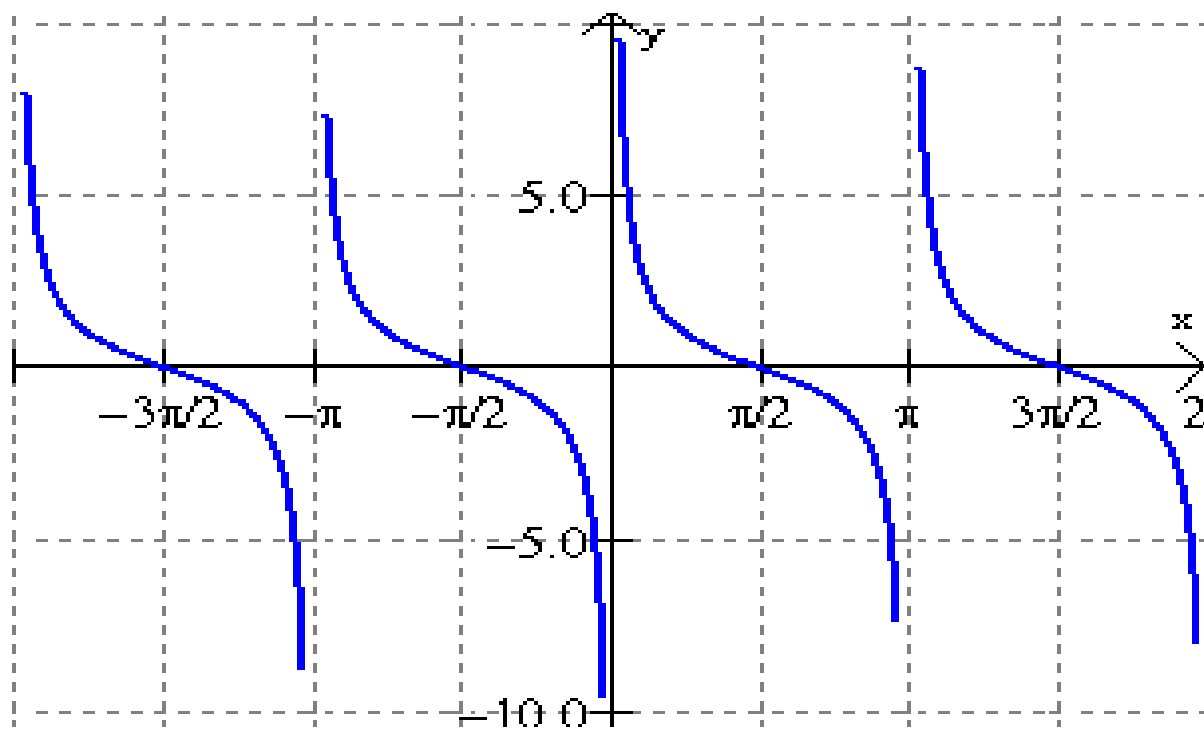
$$Y = \cos(x) \text{ چارچو}$$



# $Y = \tan(x)$ ڄاڻو



# $Y = \cotg(x)$ گراف



# مقادیر نسبت های مثلثاتی مهم

زاویه	۳۰	۴۵	۶۰
نسبت	$\pi/6$	$\pi/4$	$\pi/3$
$\sin \theta$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\cos \theta$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
$\tan \theta$	$\sqrt{3}/3$	۱	$\sqrt{3}$
$\cot \theta$	$\sqrt{3}$	۱	$\sqrt{3}/3$

# مقادیر نسبت های مثلثاتی مهم

زاویه \ نسبت	۰	۹۰	۱۸۰	۲۷۰	۳۶۰
	۰	$\pi/۲$	$\pi$	$۳\pi/۲$	$۲\pi$
$\sin \theta$	۰	۱	۰	-۱	۰
$\cos \theta$	۱	۰	-۱	۰	۱
$\tan \theta$	۰	ت.ن	۰	ت.ن	۰
$\cot \theta$	ت.ن	۰	ت.ن	۰	ت.ن

# فرمولهای مقدماتی

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \begin{cases} \sin^2 \theta = 1 - \cos^2 \theta \\ \cos^2 \theta = 1 - \sin^2 \theta \end{cases}$$

$$\tan \theta \cdot \cot \theta = 1 \Rightarrow \begin{cases} \tan \theta = \frac{1}{\cot \theta} \\ \cot \theta = \frac{1}{\tan \theta} \end{cases}$$

$$\begin{cases} \tan \theta = \frac{\sin \theta}{\cos \theta} \\ \cot \theta = \frac{\cos \theta}{\sin \theta} \end{cases}$$

$$\begin{cases} 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta} = \sec^2 \theta \\ 1 + \cot^2 \theta = \frac{1}{\sin^2 \theta} = \csc^2 \theta \end{cases}$$



# فرمولهای مجموع و تفاضل دو زاویه

$$\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan (\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} = \frac{\cot \beta \pm \cot \alpha}{\cot \alpha \cot \beta \mp 1}$$

$$\cot (\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha} = \frac{1 \mp \tan \alpha \tan \beta}{\tan \alpha \pm \tan \beta}$$

# روابط جمع به ضرب

$$\sin(a) + \sin(b) = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$\sin(a) - \sin(b) = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$$

$$\cos(a) + \cos(b) = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$\cos(a) - \cos(b) = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$$

$$\tan a \pm \tan b = \frac{\sin (a \pm b)}{\cos a \cos b}$$

$$\cot a \pm \cot b = \frac{\sin (b \pm a)}{\sin a \sin b}$$

# روابط ضرب به جمع

$$\sin a \cos b = \frac{1}{2} [\sin(a + b) + \sin(a - b)]$$

$$\sin a \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a - b) + \cos(a + b)]$$

$$\tan a \tan b = \frac{\tan a + \tan b}{\cot a + \cot b}$$

$$\cot a \cot b = \frac{\cot a + \cot b}{\tan a + \tan b}$$

# روابط دو برابر کمان

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \left\{ \begin{array}{l} = 2 \cos^2 \alpha - 1 \\ = 1 - 2 \sin^2 \alpha \\ = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} \\ = \frac{\cot^2 \alpha - 1}{\cot^2 \alpha + 1} \end{array} \right.$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2}{\cot \alpha - \tan \alpha}$$

$$\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha} = \frac{\cot \alpha - \tan \alpha}{2}$$

# روابط سه برابر قمان

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$\cot 3\alpha = \frac{3 \cot \alpha - \cot^3 \alpha}{1 - 3 \cot^2 \alpha}$$

# فرمولهای فرعی

$$(\sin x \pm \cos x)^2 = 1 \pm \sin 2x$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{\tan \alpha}{\cot \alpha}$$

$$\cot^2 \alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha} = \frac{\cot \alpha}{\tan \alpha}$$

$$\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right)$$

$$\sin x - \cos x = \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = -\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)$$

# ادامه فرمولهای فرعی

$$\frac{1 - \tan \alpha}{1 + \tan \alpha} = \tan \left( \frac{\pi}{4} - \alpha \right)$$

$$\frac{1 + \tan \alpha}{1 - \tan \alpha} = \tan \left( \frac{\pi}{4} + \alpha \right)$$

$$\tan \alpha + \cot \alpha = \frac{2}{\sin 2\alpha}$$

$$\tan \alpha - \cot \alpha = -2 \cot 2\alpha$$

$$\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha$$

# ادامه فرمولهای فرعی

$$\cos^f \alpha + \sin^f \alpha = 1 - \frac{1}{r} \sin^r 2\alpha$$

$$\cos^f \alpha - \sin^f \alpha = \cos 2\alpha$$

$$\sin^f \alpha + \cos^f \alpha = 1 - \frac{r}{f} \sin^r 2\alpha$$

$$\sin^f \alpha - \cos^f \alpha = \cos 2\alpha \left( \frac{1}{f} \sin^r 2\alpha - 1 \right)$$

$$\tan^r \alpha - \sin^r \alpha = \tan^r \alpha \cdot \sin^r \alpha$$

$$\cot^r \alpha - \cos^r \alpha = \cot^r \alpha \cdot \cos^r \alpha$$

$$\tan(a + b) - \tan a - \tan b = \tan(a + b) \tan a \tan b$$



# ادامه فرمولهای فرعی

$$\sin(\alpha - \beta) \sin(\alpha + \beta) = \cos^2 \beta - \cos^2 \alpha = \sin^2 \alpha - \sin^2 \beta$$

$$\cos(\alpha - \beta) \cos(\alpha + \beta) = \cos^2 \alpha - \sin^2 \beta$$

$$\sin \alpha \sin(60^\circ - \alpha) \sin(60^\circ + \alpha) = \frac{1}{4} \sin 3\alpha$$

$$\cos \alpha \cos(60^\circ - \alpha) \cos(60^\circ + \alpha) = \frac{1}{4} \cos 3\alpha$$

$$\tan \alpha \tan(60^\circ - \alpha) \tan(60^\circ + \alpha) = \tan 3\alpha$$

$$\cot \alpha \cot(60^\circ - \alpha) \cot(60^\circ + \alpha) = \cot 3\alpha$$

$$\tan x + \tan(x - 60^\circ) + \tan(x + 60^\circ) = 3 \tan 3\alpha$$

# ادامه فرمولهای فرعی

$$\cos a \cos 2a \dots \dots \dots \cos(2^{n-1} a) = \frac{\sin(2^n a)}{2^n \sin a}$$

$$\frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha} = \tan \frac{\alpha}{2}$$

$$\text{if } \alpha + \beta = \frac{3\pi}{4} \implies \tan \alpha + \tan \beta = \tan \alpha \tan \beta - 1$$

$$\text{if } \alpha + \beta = \frac{\pi}{4} \implies \tan \alpha + \tan \beta = 1 - \tan \alpha \tan \beta$$

# معادلات مثلثاتی

$$\sin x = \sin \alpha \implies \langle \begin{array}{l} x = \forall k\pi + \alpha \\ x = \forall k\pi + (\pi - \alpha) \end{array}$$

$$\cos x = \cos \alpha \implies \langle x = \forall k\pi \pm \alpha$$

$$\tan x = \tan \alpha \implies \langle x = k\pi + \alpha$$

$$\cot x = \cot \alpha \implies \langle x = k\pi + \alpha$$

# مالات فاص معادله مثلثاتی

$$\left. \begin{array}{l} \sin^r x = \sin^r \alpha \\ \cos^r x = \cos^r \alpha \\ \tan^r x = \tan^r \alpha \\ \cot^r x = \cot^r \alpha \end{array} \right\} \implies x = k\pi \pm \alpha$$

# مات فاص معادله مثلثاتی

$$\sin x = \cdot \implies x = k\pi$$

$$\cos x = \cdot \implies x = k\pi \pm \frac{\pi}{\gamma}$$

$$\tan x = \cdot \implies x = k\pi$$

$$\cot x = \cdot \implies x = k\pi \pm \frac{\pi}{\gamma}$$

$$\tan x = \cot x \implies x = k\pi \pm \frac{\pi}{\varphi}$$

# مات فاص معادله مثلثاتی

$$\sin x = 1 \implies x = 2k\pi + \frac{\pi}{2}$$

$$\cos x = 1 \implies x = 2k\pi$$

$$\sin x = -1 \implies x = 2k\pi + \frac{3\pi}{2} \quad \text{یا} \quad x = 2k\pi - \frac{\pi}{2}$$

$$\cos x = -1 \implies x = (2k + 1)\pi$$

$$|\sin x| = |\cos x| \implies x = k\pi \pm \frac{\pi}{4}$$

$$\sin^{2k} x = \cos^{2k} x \implies x = k\pi \pm \frac{\pi}{4}$$

# برد توابع مثلثاتی

$$-1 \leq \sin^{k-1} x \leq 1$$

$$-1 \leq \cos^{k-1} x \leq 1$$

$$-\infty < \tan x < +\infty$$

$$-\infty < \cot x < +\infty$$

$$-1 \leq \sin^k x \leq 1$$

$$-1 \leq \cos^k x \leq 1$$

# ادامه برد توابع مثلثاتی

$$-\sqrt{a^r + b^r} \leq a \sin x + b \cos x \leq \sqrt{a^r + b^r}$$

$$b \leq a \sin^r \theta + b \leq a + b \quad , \quad a > 0$$

$$b \leq a \cos^r \theta + b \leq a + b \quad , \quad a > 0$$

$$a + b \leq a \sin^r \theta + b \leq a \quad , \quad a < 0$$

$$a + b \leq a \cos^r \theta + b \leq a \quad , \quad a < 0$$

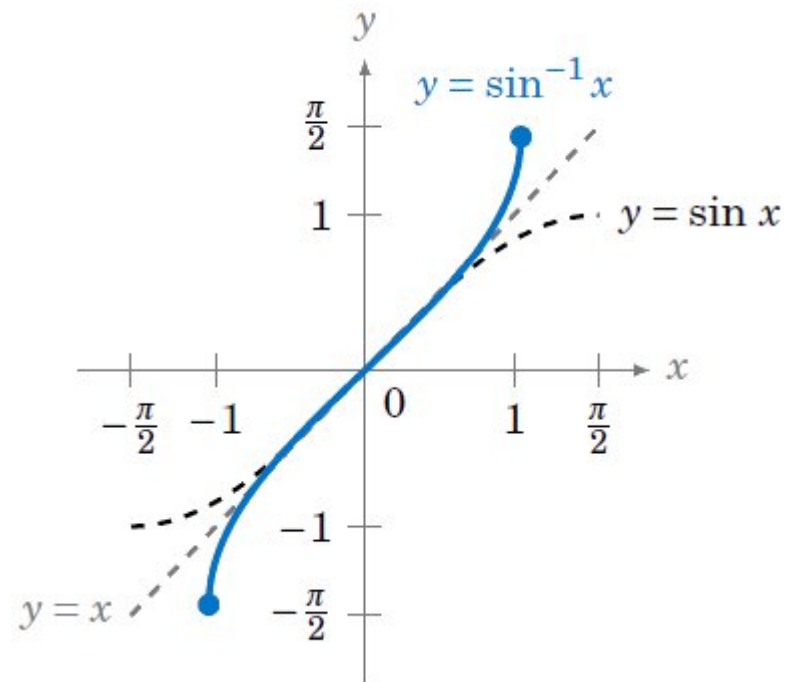
$$-(|a| + |b|) \leq a \sin x + b \cos y \leq (|a| + |b|) \quad , \quad x \neq y$$

$$r^{1-n} \leq \sin^{rn} x + \cos^{rn} x \leq 1 \quad , \quad n \in \mathcal{N}$$

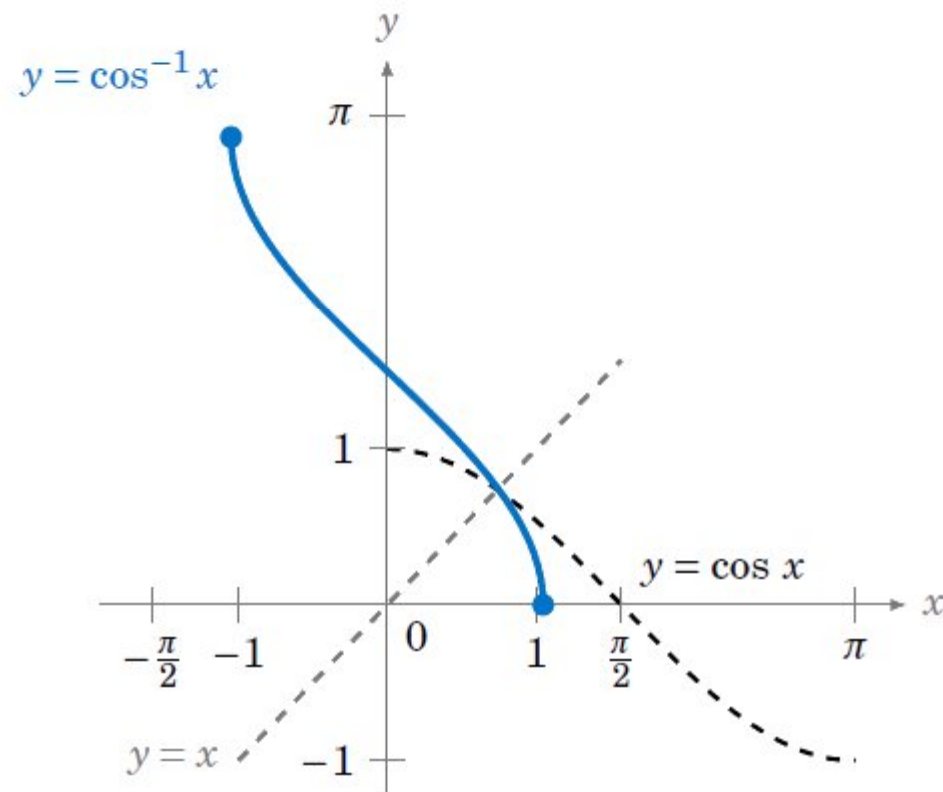
$$-1 \leq \sin^{r(n+1)} x + \cos^{r(n+1)} x \leq 1 \quad , \quad n \in \mathcal{N}$$



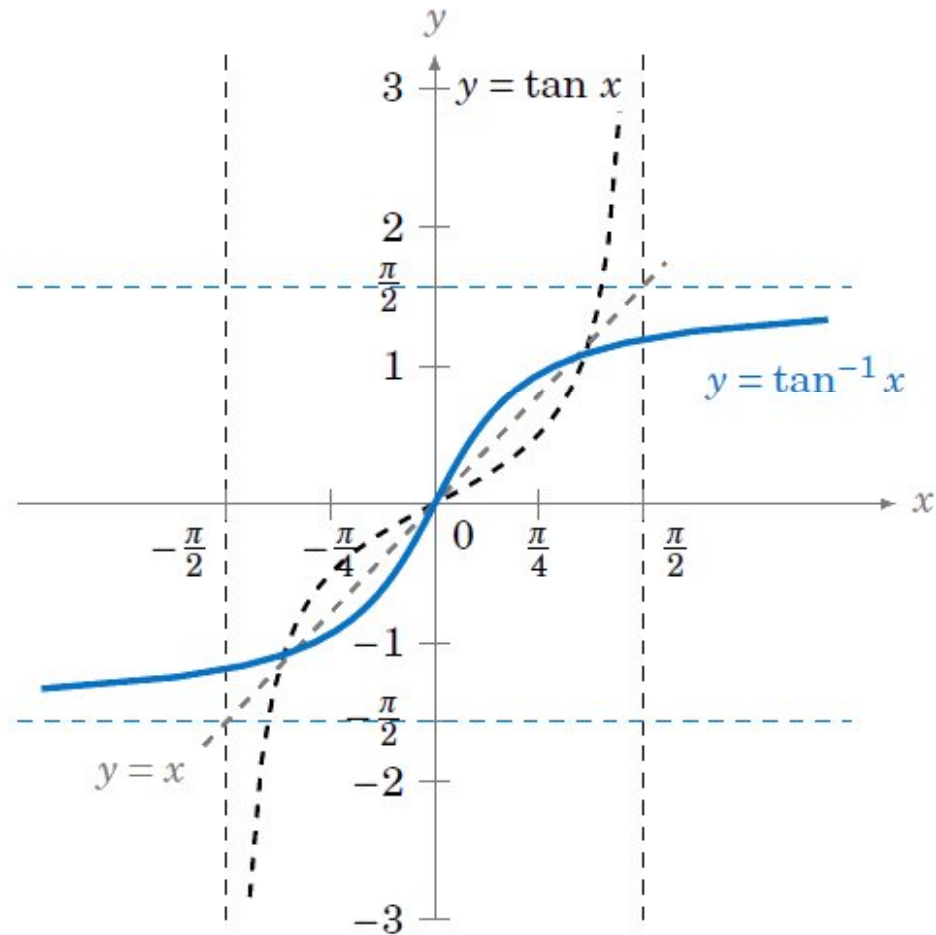
# $Y = \text{Arcsin}(x)$ ډاډه



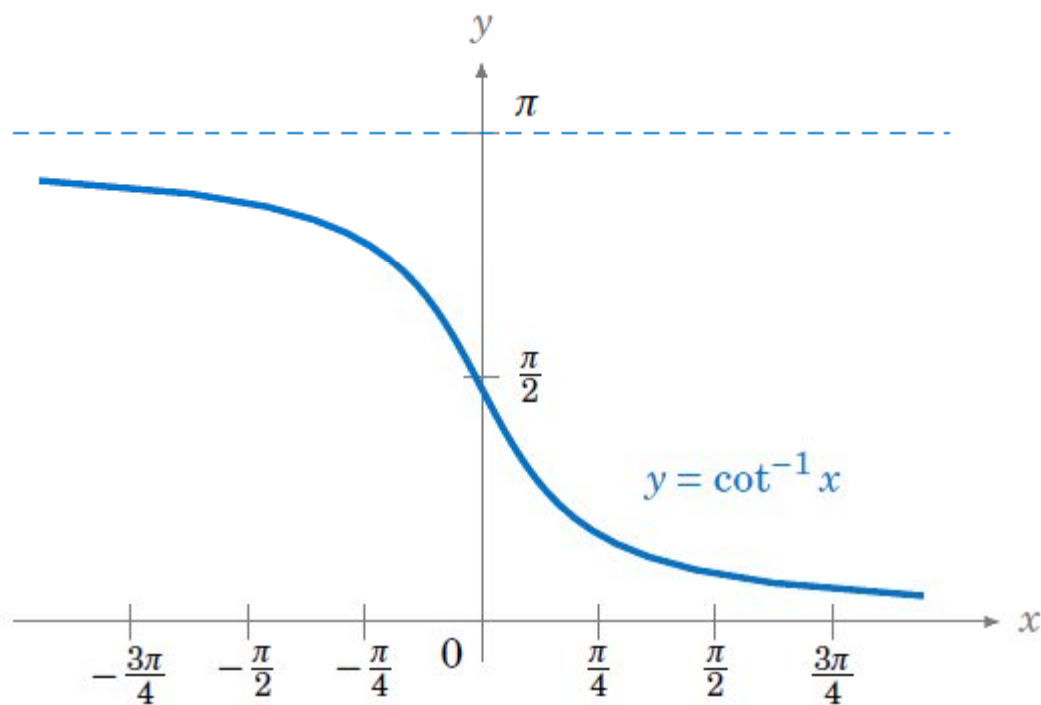
# $Y = \text{Arccos}(x)$ ډډه



# $Y = \text{Arctg} x$ ډډه



# $Y = \text{Arccotg}(x)$ ډډډډ



# توابع معکوس مثلثاتی

$$y = \sin x \implies x = \text{Arc sin } y \quad , \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad , \quad y \in [-1, 1]$$

$$y = \cos x \implies x = \text{Arc cos } y \quad , \quad x \in [0, \pi] \quad , \quad y \in [-1, 1]$$

$$y = \tan x \implies x = \text{Arc tan } y \quad , \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \quad , \quad y \in \mathcal{R}$$

$$y = \cot x \implies x = \text{Arc cot } y \quad , \quad x \in (0, \pi) \quad , \quad y \in \mathcal{R}$$

# ویژگی‌های توابع آرک

$$\text{Arc sin}(-x) = - \text{Arc sin } x$$

$$\text{Arc cos}(-x) = \pi - \text{Arc cos } x$$

$$\text{Arc tan}(-x) = - \text{Arc tan } x$$

$$\text{Arc cot}(-x) = \pi - \text{Arc cot } x$$

# ویژگی‌های توابع آرک

$$\text{Arc sin}(\sin x) = \text{Arc cos}(\cos x) = x$$

$$\sin(\text{Arc sin } x) = \cos(\text{Arc cos } x) = x \quad , \quad |x| \leq 1$$

$$\tan(\text{Arc tan } x) = \cot(\text{Arc cot } x) = x \quad , \quad x \in \mathcal{R}$$

$$\sin(\text{Arc cos } x) = \cos(\text{Arc sin } x) = \sqrt{1 - x^2} \quad , \quad |x| \leq 1$$

$$\tan(\text{Arc cot } x) = \cot(\text{Arc tan } x) = \frac{1}{x} \quad , \quad x \in \mathcal{R}$$

# ویژگی های توابع آرک

$$\text{Arc sin } x = \text{Arc cos } \sqrt{1 - x^2} \quad , \quad |x| \leq 1$$

$$\text{Arc cos } x = \text{Arc sin } \sqrt{1 - x^2} \quad , \quad |x| \leq 1$$

$$\text{Arc tan } x = \begin{cases} \text{Arc cot } \frac{1}{x} & , \quad x > 0 \\ \pi - \text{Arc cot } \frac{1}{x} & , \quad x < 0 \end{cases}$$

$$\text{Arc cot } x = \begin{cases} \text{Arc tan } \frac{1}{x} & , \quad x > 0 \\ \pi + \text{Arc tan } \frac{1}{x} & , \quad x < 0 \end{cases}$$



# ویژگی های توابع آرک

$$\text{Arc cos } x = \text{Arc tan } \frac{\sqrt{1-x^2}}{x}, \quad 0 \leq x \leq 1$$

$$\text{Arc tan } x = \text{Arc cos } \frac{1}{\sqrt{1+x^2}}, \quad x \geq 0$$

$$\text{Arc sin } x = \text{Arc tan } \frac{x}{\sqrt{1-x^2}}, \quad -1 \leq x \leq 1$$

$$\text{Arc sin } x + \text{Arc cos } x = \frac{\pi}{2}, \quad |x| \leq 1$$

$$\text{Arc tan } x + \text{Arc cot } x = \frac{\pi}{2}, \quad x \in \mathcal{R}$$

# ویژگی های توابع آرک

$$\text{Arc tan } x + \text{Arc tan } \frac{1}{x} = \begin{cases} \frac{\pi}{2} & , \quad x > 0 \\ -\frac{\pi}{2} & , \quad x < 0 \end{cases}$$

$$\text{Arc cot } x + \text{Arc cot } \frac{1}{x} = \begin{cases} \frac{\pi}{2} & , \quad x > 0 \\ \frac{3\pi}{2} & , \quad x < 0 \end{cases}$$

$$\text{Arc tan } x + \text{Arc tan } y = \text{Arc tan } \frac{x + y}{1 - xy}$$

$$\text{Arc tan } x - \text{Arc tan } y = \text{Arc tan } \frac{x - y}{1 + xy}$$